Discovery : Jurnal Ilmu Pengetahuan
Volume 10 (1) 1 – 31 March 2025
ISSN: 2527-6859 (Print) / ISSN: 2723-6145 (Online)
The article is published with Open Access at: https://ejournal.unhasy.ac.id/index.php/discovery/index

Connecting Theorems and Definitions: Students' Exploration of Function Derivatives

Zainullah Zuhri*, Universitas Nahdlatul Ulama Pasuruan

zuhri@unupasuruan.ac.id

Abstract: This study aims to analyze students' understanding of derivatives of functions through independent exploration, where students were asked to create examples of functions, calculate their derivatives using differentiation theorems, and prove the results using the definition of derivatives. The subjects of the study were 35 first-semester students from the mathematics education program. Data was collected through exploratory tasks, interviews, and observations. The results showed that all students were able to determine the derivatives of the functions they created, although proving the derivative using the definition was more challenging as it involved the concept of limits previously studied. Five students demonstrated a deeper understanding by creating more complex examples, such as root and rational functions, which required manipulation techniques and the application of the quotient rule for derivatives. Interviews revealed that proving the derivative with the definition provided a deeper understanding of the fundamental concepts of derivatives. This study suggests using varied exploratory tasks and reflection sessions to enhance students' conceptual understanding of derivatives.
Keywords: Calculus learning strategies, Differentiation theorems, Function derivatives

Received : February 4^{th} 2025; **Accepted : March 11^{th} 2025** ; **Published : March 12^{th} 2025**

Citation: Zuhri, Z. (2025). Connecting Theorems and Definitions: Students' Exploration of Function Derivatives. *Discovery : Jurnal Ilmu Pengetahuan*, *10* (1), 13 – 19, https://doi.org/10.33752/jd.v09i2.8615

INTRODUCTION

Calculus, as a fundamental branch of mathematics, plays a significant role in understanding change and rates of change, with derivatives being one of its core concepts. The derivative not only forms the basis for solving various real-world problems but also serves as a foundational concept for advanced mathematical studies. A solid understanding of the basic principles of derivatives is, therefore, essential for students pursuing mathematics or related disciplines (Byerley, 2019; Haghjoo et al, 2020; Mkhatshwa, 2020; Gunarti dkk, 2022, Santos-Trigo et al, 2024; Tarasov, 2019).

However, despite its importance, many students face challenges in understanding derivatives conceptually. These challenges often stem from a lack of clarity in connecting the formal definition of a derivative, which is grounded in limits, to the differentiation theorems commonly used for computation. This disconnects leads students to rely heavily on procedural methods, such as applying differentiation rules, without fully grasping the underlying principles.

Mathematics, in general, presents significant difficulties for students, particularly in transitioning from computational proficiency to conceptual understanding (Ikhsan & Sa'adah, 2024; Lailiyah & Zuhri, 2024; Sriwijayati et al., 2024; Zahro & Zuhri, 2024). Many students perceive mathematical concepts as abstract and struggle with bridging symbolic representations to their real-world applications. This difficulty is further exacerbated in calculus, where students must not only compute derivatives but also understand their meaning within different contexts. A key factor in overcoming these challenges is fostering critical thinking, which enables students to analyze problems beyond procedural applications and develop a deeper comprehension of mathematical reasoning.

Previous studies have highlighted the difficulties students encounter in comprehending the formal definition of a derivative. For instance, the cognitive transitions required to move between different representations of derivatives, such as graphical, numerical, and symbolic (Ikram et al, 2020; Taniguchi et al, 2018). While these studies have contributed to understanding students' challenges, most have focused on analyzing student misconceptions without actively engaging them in constructing and validating derivatives independently (Miswaro & Zuhri, 2023; Watford, 2024; Zuhri, 2024).

The gap between procedural knowledge and conceptual understanding is particularly evident when students attempt to justify their work. Without actively engaging in derivations and proofs, they may develop fragmented knowledge, where they apply differentiation rules correctly but struggle to explain why they work. Encouraging students to derive results from first principles can help address this issue by reinforcing connections between fundamental concepts and computational techniques.

This study offers a novel approach by involving students in the process of creating their own functions, determining derivatives using differentiation theorems, and verifying their results through the formal limit definition. Unlike previous research that primarily investigates existing misconceptions, this study emphasizes independent exploration as a method for reinforcing conceptual understanding. By doing so, it aims to uncover how this process helps students bridge the gap between theorems and definitions while identifying the specific challenges they face in this context.

Furthermore, this study highlights the role of critical thinking in mathematical learning. By requiring students to justify their reasoning and verify their results independently, the research explores how engagement with first-principal definitions can foster deeper understanding. This perspective aligns with modern pedagogical approaches that emphasize active learning and conceptual inquiry over rote memorization.

This study offers a novel approach by involving students in the process of creating their own functions, determining derivatives using differentiation theorems, and verifying their results through the formal limit definition. Unlike previous research that primarily investigates existing misconceptions, this study emphasizes independent exploration as a method for reinforcing conceptual understanding. By doing so, it aims to uncover how this process helps students bridge the gap between theorems and definitions while identifying the specific challenges they face in this context.

METHODS

This study employed a qualitative approach with a descriptive method to explore how students connect differentiation theorems with the formal definition of a derivative. The participants consisted of 35 first-semester students from the 2024 cohort of a mathematics education program. These students, at the beginning of their calculus learning journey, were deemed suitable for investigating their foundational understanding of derivatives.

The primary instrument used in this study was an exploratory task, where students were asked to create their own functions, determine the derivatives using differentiation theorems, and verify their results through the formal limit definition of a derivative. To complement this, follow-up interviews were conducted with selected students who provided varied responses during the task. The selection criteria were based on the diversity of their approaches and the level of difficulty they encountered while solving the problems. This selection aimed to gain deeper insights into different levels of understanding and the specific challenges students faced in connecting the theorem-based and definition-based approaches. Additionally, the researcher observed students during the completion of the tasks to document their strategies and approaches.

Prior to participation, students provided informed consent, ensuring their voluntary involvement in the study. Confidentiality was maintained by anonymizing their responses and using pseudonyms where necessary to protect their identities. The procedure began with an initial explanation of the task and guidelines to ensure students understand the requirements. Each student then worked individually on the exploratory task, and their work was documented for further analysis. Finally, selected students were interviewed to explore their understanding and reflections in greater depth, providing valuable data for analysis.

RESULTS and DISCUSSION

To make it easier to display research results and discussions, the following points are made in the form of points

Complexity of Functions Created by Students

All students involved in this study successfully determined the derivative of the functions they created. While they did not face difficulty applying differentiation theorems to calculate derivatives, proving the results using the definition of the derivative proved more challenging. This proof process required the use of the concept of limits, which had been studied previously. Although more demanding, students were able to correctly complete the proof, demonstrating their ability to connect theorems with fundamental calculus concepts, even though it required extra effort to understand the application of the limit definition in the context of derivatives. This finding aligns with previous research (Alam, 2020) which emphasizes the critical role of limits in understanding the derivative concept.

Complex Examples: Root and Rational Functions

After further analysis, it was found that 5 students created more complex examples of functions compared to the other 30 students. These more complex functions included forms such as root functions and rational functions. Proving the derivative of a root function using the definition (through the concept of limits) required a specific strategy. This strategy involved manipulating the root exponent into a fractional power, which presented an additional challenge. On the other hand, deriving rational functions demanded a more complex approach, requiring the application of the quotient rule for derivatives. Some of the students' work can be seen in Figure 1 and Figure 2 below. This finding aligns with studies suggesting that students who work with more complex functions tend to experience more cognitive load, which could influence their problem-solving strategies (Kirschner et al, 2018; Tharp, D. S, 2018).

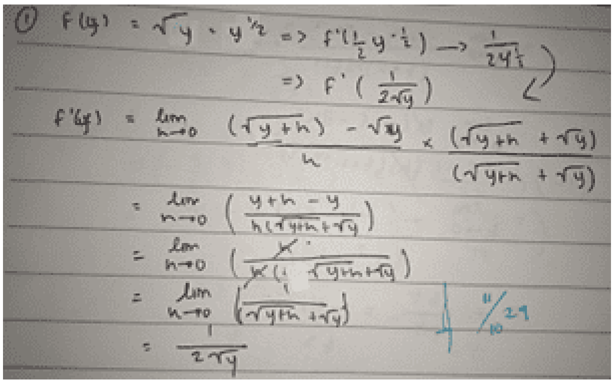

FIGURE 1. E*xample of root function*

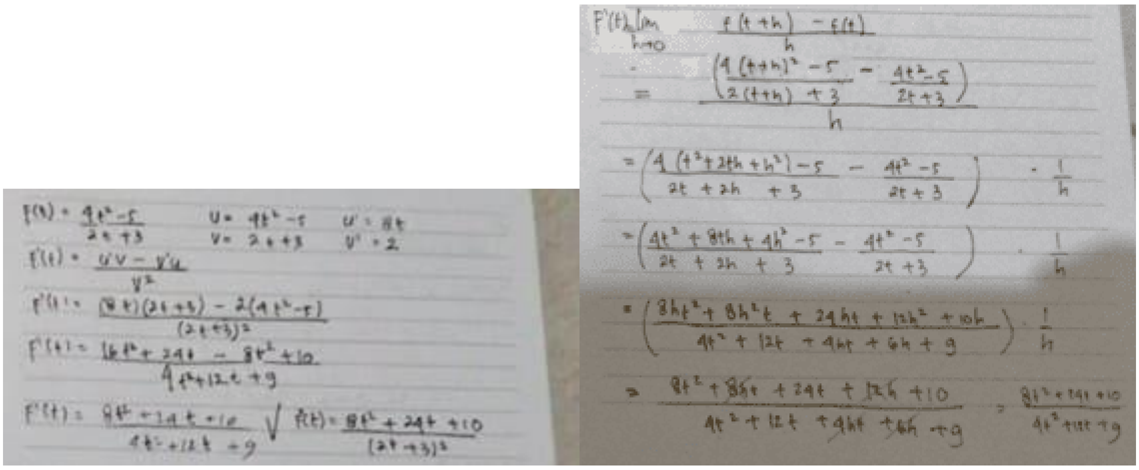

FIGURE 2. E*xample of rational function*

Insights from Interviews

Based on the interviews, students who produced work with varying results expressed that using the definition of the derivative required a deeper understanding compared to simply applying differentiation theorems. DS admitted, "At first, I thought differentiation was just about using formulas, but when I had to prove it with the definition, I realized that I didn’t fully understand the core concept. I had to think more deeply about limits and how small changes in x affect the function." Similarly, FZ stated, "When we use differentiation rules, we get the answer immediately. But with the definition, I had to understand why the result is correct rather than just memorizing formulas." They also noted that the process of proving derivatives using the definition helped them better grasp the connection between calculation procedures and the fundamental concept of derivatives. RF shared, "When I tried to prove the derivative of a root function using the definition, I had to first rewrite it as an exponent. That made me more aware of how limits actually work in differentiation."

DK added, "Before this task, I saw differentiation as just a set of formulas. But after proving it using the definition, I started to understand that derivatives are really about instantaneous rates of change and how limits are used to determine them." These insights support the theory that conceptual understanding is enhanced when students are required to justify their answers rather than simply applying formulas. (Loibl, K. et al 2017; Lomibao, L. S. et al, 2016; Rosé, C. P. et al, 2019).

Insights from Observations

From the observations during the task execution, students with a stronger conceptual understanding were generally more confident in applying the definition of the derivative. They appeared to be more systematic and careful in completing the proof, in contrast to students who frequently experienced confusion or asked for clarification regarding the application of the definition of the derivative. This observation is consistent with research indicating that students with a solid conceptual base are more likely to demonstrate greater confidence in mathematical tasks, particularly in those requiring justification and proof (Lockwood, E. et al, 2016; Nathan, M. J., & Walkington, C., 2017; Stylianides, A. J. et al, 2016).

CONCLUSION

This study shows that all students were able to determine the derivatives of the functions they created, although proving the derivative using the definition was more challenging and required a deeper understanding of the concept of limits. Students with a stronger conceptual understanding were more systematic in completing the proof. Students who created more complex examples, such as root and rational functions, faced greater challenges in proving the derivatives. Based on interviews, they felt that using the definition of the derivative deepened their understanding. Therefore, it is recommended to emphasize foundational conceptual understanding through varied exploratory tasks, reflection sessions, and additional learning aids.

REFERENCES

1. Alam, A. (2020). Challenges and possibilities in teaching and learning of calculus: A case study of India. *Journal for the Education of Gifted Young Scientists*, *8*(1), 407-433.
2. Byerley, C. (2019). Calculus students’ fraction and measure schemes and implications for teaching rate of change functions conceptually. *The Journal of Mathematical Behavior*, *55*, 100694.
3. Haghjoo, S., Reyhani, E., & Kolahdouz, F. (2020). Evaluating the understanding of the university students (Basic Sciences and Engineering) about the numerical representation of the average rate of change. *International Journal of Educational and Pedagogical Sciences*, *14*(2), 111-121.
4. Ikhsan, M. M., & Sa’adah, N. (2024). Pengaruh Model Pembelajaran Kooperatif Tipe Stad Terhadap Prestasi Belajar Matematika Siswa Sma. *Discovery : Jurnal Ilmu Pengetahuan*, *9*(2), 85–94. https://doi.org/10.53565/pssa.v6i2.180
5. Ikram, M., Purwanto, P., Parta, I. N., & Susanto, H. (2020). Mathematical reasoning required when students seek the original graph from a derivative graph. *Acta Scientiae*, *22*(6), 45-64.
6. Kirschner, P. A., Sweller, J., Kirschner, F., & Zambrano R, J. (2018). From cognitive load theory to collaborative cognitive load theory. International journal of computer-supported collaborative learning, 13, 213-233.

7. Lailiyah, N., & Zuhri, Z. (2024). The Effectiveness of the Application M3 Board Learning Media (Mean, Median, and Mode) in Class VIII Statistics Materials at Lekok NU Middle School. *Noumerico: Journal of Technology in Mathematics Education*, *2*(1), 14-22.
8. Lockwood, E., Ellis, A. B., & Lynch, A. G. (2016). Mathematicians' example-related activity when exploring and proving conjectures. International Journal of Research in Undergraduate Mathematics Education, 2, 165-196.
9. Loibl, K., Roll, I., & Rummel, N. (2017). Towards a theory of when and how problem solving followed by instruction supports learning. *Educational psychology review*, *29*, 693-715
10. Lomibao, L. S., Luna, C. A., & Namoco, R. A. (2016). The influence of mathematical communication on students' mathematics performance and anxiety. American Journal of Educational Research, 4(5), 378-382
11. Miswaro, S. M., & Zuhri, Z. (2023). ANALISIS KESULITAN SISWA DALAM MENYELESAIKAN SOAL CERITA MATEMATIKA PADA MATERI STATISTIKA DITINJAU DARI KEMAMPUAN MATEMATIKA SISWA. *Center of Education Journal (CEJou)*, *4*(2).
12. Mkhatshwa, T. P. (2020). Calculus students' quantitative reasoning in the context of solving related rates of change problems. *Mathematical Thinking and Learning*, *22*(2), 139-161
13. Nathan, M. J., & Walkington, C. (2017). Grounded and embodied mathematical cognition: Promoting mathematical insight and proof using action and language. *Cognitive research: principles and implications*, *2*, 1-20
14. Nathan, M. J., & Walkington, C. (2017). Grounded and embodied mathematical cognition: Promoting mathematical insight and proof using action and language. *Cognitive research: principles and implications*, *2*, 1-20
15. Rosé, C. P., McLaughlin, E. A., Liu, R., & Koedinger, K. R. (2019). Explanatory learner models: Why machine learning (alone) is not the answer. *British Journal of Educational Technology*, *50*(6), 2943-2958
16. Santos-Trigo, M., Camacho-Machín, M., & Barrera-Mora, F. (2024). Focusing on foundational Calculus ideas to understand the derivative concept via problem-solving tasks that involve the use of a dynamic geometry system. *ZDM–Mathematics Education*, 1-15
17. Sriwijayati, A. M., Febriyanti, R., & Albab, M. U. (2024). Pengaruh Model Pembelajaran Berbasis Masalah Berbantuan Geogebra Terhadap Kemampuan Berpikir Kritis Siswa Smp. *Discovery : Jurnal Ilmu Pengetahuan*, *9*(2), 122–130. https://doi.org/10.36928/jsm.v6i2.2813
18. Stylianides, A. J., Bieda, K. N., & Morselli, F. (2016). Proof and argumentation in mathematics education research. In The second handbook of research on the psychology of mathematics education (pp. 315-351). Brill
19. Taniguchi, T., Ugur, E., Hoffmann, M., Jamone, L., Nagai, T., Rosman, B., ... & Wörgötter, F. (2018). Symbol emergence in cognitive developmental systems: a survey. IEEE transactions on Cognitive and Developmental Systems, 11(4), 494-516.

20. Tarasov, V. E. (2019). On history of mathematical economics: Application of fractional calculus. *Mathematics*, *7*(6), 509
21. Tharp, D. S. (2018). Variations among White First-Year College Students' Understanding of Privilege (Doctoral dissertation, University of Illinois at Chicago).
22. Watford Jr, J. M. (2024). *A Commognitive Analysis of Two Discursive Learning Environments: Exploring Students' Discourse Regarding the Derivative Concept* (Doctoral dissertation, The Florida State University)
23. Zahro, S. F., & Zuhri, Z. (2024). Analisis Kemampuan Berpikir Kritis Siswa dalam Menyelesaikan Masalah Materi Program Linear Ditinjau dari Gaya Kognitif Field-Dependent dan Field-Independent. *Discovery : Jurnal Ilmu Pengetahuan*, *09*(October), 111–121.
24. Zuhri, Z. (2024). EKSPLORASI TURUNAN FUNGSI: DUKUNGAN AI DALAM MENJEMBATANI TEOREMA DAN DEFINISI. *Center of Education Journal (CEJou)*, *5*(02).
25. Gunarti, M. R., Harini, N. V., Zuhri, Z., & Dharmawan, P. B. (2022, December). Interpreting kinematics graph: What do students' show?. In *AIP Conference Proceedings* (Vol. 2468, No. 1). AIP Publishing.

PROFILE

Zainullah Zuhri is a lecturer on department of mathematics education Universitas Nahdlatul Ulama Pasuruan. He actives on several research project especially in the field of Psychology of mathematics learning.